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MEMORANDUM REPORT NO. 2211

OPTIMUM PROJECTILE SHAPE FOR IMPROVED AMMUNITION

by

Joseph H. Spurk
Nathan Gerber

August 1972

U.S. ARMY ABERDEEN RESEARCH AND DEVELOPMENT CENTER
BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

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B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

MEMORANDUM REPORT NO. 2211

AUGUST 1972

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Joseph H. Spurk
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Exterior Ballistics Laboratory

RDT&E Project No. 1T061102A33D

A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

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MEMORANDUM REPORT NO. 2211

JHSpurk & NGerber/eb
Aberdeen Proving Ground, Md.
August 1972

OPTIMUM PROJECTILE SHAPE FOR IMPROVED AMMUNITION

ABSTRACT

A study is made to determine the optimum shape of an artillery shell used to deliver improved ammunition. The "optimum shape" here is the one that minimizes the dead mass of the shell for a given payload; equivalently the problem is to minimize the surface of a shell with specified volume and total drag coefficient. Linear aerodynamics and several simplifying assumptions are used in stating the drag coefficient dependence on body shape and flight parameters. Restricting the scope of the problem to specific families of curves greatly simplifies the calculations and yet provides valuable guidance in determining optimum shapes; in particular, cone-cylinders and parabolic tangent-ogives are studied. Diagrams are presented showing the dependence of surface area, for unit volume, on the parameters δ (nose thickness-ratio) and k (nose percentage of projectile length). It is found that optimum shapes do not have large length-to-diameter ratios and are nearly independent of Mach number in supersonic flight.

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LIST OF SYMBOLS

A	$SV^{-2/3}$
c_1	constant used in evaluating head drag ($c_{DH} = c_1 \delta^2$)
C_f	skin friction coefficient, Eq. (3)
C_{DB}	base pressure drag coefficient
C_{DF}	friction drag coefficient
C_{DH}	head (nose) pressure drag coefficient
C_{Do}	total drag coefficient of projectile, Eq. (1)
d	diameter of projectile, Figure 1
F	$F(\delta, k) = 0$ -- drag coefficient constraint, Eqs. (6) and (10)
k	nose percentage of projectile length, Figure 1
K_1, K_2	constants in Eq. (2)
ℓ	length of projectile
L	length of gun barrel
M_∞	flight Mach number
$\bar{p}$	average pressure in gun
q	$= \rho_\infty U_\infty{}^2/2$
r	$= r(x)$, radial coordinate of point on projectile surface
R	projectile trajectory range
Re	Reynolds number based on ℓ, ρ_∞, U_∞, and viscosity of the undisturbed air
S	total surface area of projectile
$\overset{*}{S}$	$= S - (\pi d^2/4)$
U_∞	flight speed of projectile
V	volume of projectile
x	axial coordinate

LIST OF SYMBOLS

δ	nose thickness-ratio, Figure 1
λ	Lagrangian multiplier
ρ_∞	density of the undisturbed air

I. INTRODUCTION

Artillery systems are currently being considered for the delivery of improved ammunition. In this application, the shell serves only as a container for sub-ammunition; also a small terminal velocity is desirable. The question arises: is there an "optimum" shell for this application?

Optimization of shell design is a much pursued subject. Usually a shell is considered "optimum" in an aerodynamic sense of having the smallest drag for a given volume or given length. While the consideration of drag is always of importance in ballistics, it is of primary importance only for missiles with high terminal velocity or large range.

It is proposed here that minimizing the mass of that part of the shell which is not effective in terminal ballistics is of primary importance for improved ammunition. This requires us to reduce the "dead" mass of the shell itself, which, as mentioned above, serves only as a container. There is the temptation to increase the warhead volume by simply lengthening the shell; this, however, will increase the dead mass at least at the same rate as the usable warhead volume. While lengthening the shell is, of course, the only way to increase volume for a given diameter weapon, it is not optimizing in the sense discussed above. The large ℓ/d shapes* are also difficult to spin-stabilize, and this renders their use even more difficult. It is worth pointing out that minimum dead mass not only reduces the fabrication and logistics costs, but also reduces the weight of the total artillery delivery system, since the size of the recoil mechanism is very sensitive to projectile weight.

On practical grounds, it must be acknowledged that there are essentially only three basic diameters at the disposal of the designer, namely, those of most of the existing inventory of gun barrels. However, further variation of shell diameter could be accomplished by use of sabots.

**Definitions of symbols are found in LIST OF SYMBOLS, page 9*

The wall thickness of the shell is expected to be small compared to the diameter; then the mass of the shell container is proportional to the surface, and the problem is to find the shell shape that has the smallest surface for a given volume. It is also necessary to specify the drag coefficient; this, of course, rules out the trivial shape (sphere).

If the terminal kinetic energy of the shell is negligible, the appropriate constraint is not the drag but the drag coefficient, which for a given range R and velocity U_∞ must be no larger than

$$C_{Do} = p\ L\ (Rq)^{-1}$$

and is independent of the bore area for a full caliber shell. Here p is an average pressure in the gun, L the length of the barrel, and $q \equiv \rho_\infty U_\infty^2/2$. Given the free stream conditions, the drag depends only on the body shape, as does the surface area and the volume.

The problem of minimizing the surface area under the constraints of a specified drag and volume constitutes a variational problem with isoperimetric constraints. The problem generally can only be solved with the aid of a computer; for axisymmetric missiles, for example, the result will be an "optimum" meridional curve. However, a survey of the major dimensions of such an optimum shape may be obtained by considering only families of shapes; consequently, we shall restrict ourselves to cone-cylinders and parabolic nose-cylinders (tangent ogive), shown in Figure 1.

II. DRAG COEFFICIENT

A. Assumptions

We shall assume that the major portion of the trajectory is supersonic. This assumption is not necessarily in conflict with the assumption of small terminal kinetic energy; in a more sophisticated approach

one would, of course, properly weight the Mach and Reynolds number ranges of the trajectory. We shall further restrict ourselves to slender bodies. These assumptions are not at all essential for the development of the subject, but they allow the use of closed-form expressions for the pressure drag.

The drag coefficient C_{Do} may be considered as the sum of the coefficients of various components: i) the head pressure drag coefficient C_{DH}, ii) the base pressure drag coefficient C_{DB}, and iii) the friction drag coefficient C_{DF}. Thus

$$C_{Do} = C_{DH} + C_{DB} + C_{DF} \tag{1}$$

The head pressure drag coefficient in linearized supersonic flow theory may be expressed as

$$C_{DH} = K_1 \delta^2 + K_2 \{ r\ (dr/dx) \}^2_{x=k\ell} \ \ell n[(M_\infty^2-1)^{1/2}] \tag{2}$$

where the constants (K_1, K_2) are determined solely by the value of C_{DH} for $M_\infty = 2^{1/2}$. It is seen that C_{DH} is completely independent of Mach number if $dr/dx = 0$ at the shoulder and for finite dr/dx at the shoulder, C_{DH} is only a weak function of the Mach number.

The base pressure drag coefficient depends on the Reynolds number the Mach number and the geometry. The theoretical prediction of the base pressure coefficient is not yet possible. For Reynolds numbers greater than 10^6, reference 1 reports experimental investigations of the base pressure for cone-cylinder models in a free-flight range. For these large Reynolds numbers, the base pressure appears to be independent of Reynolds number. We shall use the empirical data of reference 1; i.e.,

$$C_{DB} = \text{fn}\ (M_\infty),$$

for both cone-cylinder and tangent-ogive shapes.

The computation of the boundary layer and the friction drag on a spinning shell is also impossible without rather stringent simplification. For the present exercise, it may be sufficient to use an empirical formula which has been used in free flight and wind tunnel work to estimate the friction drag. Accordingly, the skin friction is computed on the basis of the turbulent incompressible skin friction over a flat plate having the same area as the wetted area of the projectile (exclusive of base area). The use of the incompressible skin friction coefficient in the above-mentioned application is somewhat suspect, since the friction coefficient is known to decrease due to compressibility effects. However, the use of the flat plate formula in this case is at best a rough approximation.

If C_f is the friction coefficient for the flat plate referenced to the wetted area, given by the Prandtl-Schlichting[2] formula

$$C_f = 0.455\ (\log_{10} Re)^{-2.58} - 1700\ Re^{-1}, \tag{3}$$

then the friction coefficient C_{DF} for the projectile is

$$C_{DF} = [4/(\pi d^2)]\ C_f\ \overset{*}{S},$$

where the Reynolds number is based on the total projectile length, and $\overset{*}{S}$ is the surface of the nose plus cylinder.

B. Values of the Constants

For the head pressure drag coefficient, the following approximation was made:

$$C_{DH} = c_1\ \delta^2$$

A value of $c_1 = 0.81$ was used for cone-cylinders, based on data in references 3 and 4; a value of $c_1 = K_1 = 1.2$, from Eq. (2), was used for parabolic-nose shells.

The value of $C_f = .00310$, based on $Re = 3 \times 10^6$, was obtained from Eq. (3).

The following table shows the Mach number dependence of C_{DB}:[1]

M_∞ --	1.5	2.0	2.5	3.0
C_{DB} --	.1900	.1460	.1140	.0952

For both the cone-cylinders and the parabolic-nose shells, the following set of nominal values for total drag coefficient was prescribed, based on cone-cylinder measurements reported in reference 1:

M_∞ --	1.5	2.0	2.5	3.0
C_{Do} --	.349	.293	.255	.221

III. CONE-CYLINDER

A. Optimal Projectile with Prescribed Drag Coefficient

The cone-cylinder (Figure 1a) has the surface area

$$S = \pi(k\ell)^2 \ (\delta/2) \ [\{1 + (\delta^2/4)\}^{1/2} + (\delta/2) + 2k^{-1} - 2] \tag{4}$$

and the volume

$$V = \pi(\delta^2/4) \ (k\ell)^3 \ [k^{-1} - 2/3] \tag{5}$$

The drag coefficient constraint is now

$$F \equiv -C_{Do} + c_1\delta^2 + (2C_f/\delta)[\{1 + (\delta^2/4)\}^{1/2} + 2k^{-1} - 2] + C_{DB} = 0 \tag{6}$$

It is convenient here to eliminate ℓ from the problem by taking $A \equiv SV^{-2/3}$ as the quantity to be minimized. The problem is solved, according to Lagrange's method of auxiliary multipliers, by determining the δ, k, and λ values that satisfy the following two equations plus the constraint relation, Eq. (6):

$$\begin{aligned} \partial\ [A + \lambda F]/\partial\delta &= 0 \\ \partial\ [A + \lambda F]/\partial k &= 0, \end{aligned} \tag{7}$$

where λ is a Lagrange multiplier.

The parameter k must be less than unity for a real projectile. This requirement and the drag coefficient constraint place limitations on the conditions under which solutions can exist (within the framework of the assumptions). Thus in Figure 2a, the dotted curve demonstrates that no cone-cylinder satisfying the constraint is possible if an unreasonably low C_{Do} is prescribed for the projectile. The solid curve shows the ranges of δ and k in which a solution might be found for the prescribed C_{Do} (= .255).

Figure 2b shows the variation of A with δ for M_∞ = 2.5. Figures 3 and 4 present a set of cone-cylinders of unit volume, including the optimal case, that fulfill the condition of Eq. (6). In addition, Figure 4 contains a magnified picture of the behavior of A near the minimum point. The optimum cone-cylinder has a small cylindrical section and may not be stabilizable by spin. For practical purposes, however, the shell labeled #2 will be satisfactory, for the length of the cylindrical section is increased by a factor of 1.68 over that for the optimum shell, while A is multiplied by only 1.005.

For the prescribed set of C_{Do} values, the shape of the optimum cone-cylinder was found to be only weakly dependent on Mach number, as the values in the following table indicate:

M_∞	δ	k	$(A)_{min}$
1.5	.412	.837	6.14
2.0	.393	.846	6.20
2.5	.376	.863	6.25
3.0	.356	.868	6.31

Since the dotted curve in Figure 2a indicates an unrealistic value of C_{Do}, it would be instructive to determine what values of C_{Do} are attainable under the assumptions made here. Thus, in Figure 5, C_{Do} is

plotted against δ, by Eq. (6), for the limiting value $k = 1$; the curves clearly show the lower limits to the total drag coefficients.

B. Optimal Projectile with Zero Drag

When the gasdynamic factors are neglected, i.e., when zero drag is assumed, the problem becomes a purely geometrical one. The minimal surface is found by solving the following equations for δ and k:

$$\partial A/\partial \delta = 0 \qquad\qquad \partial A/\partial k = 0$$

A series of cone-cylinders of unit volume, including the optimal case, satisfying the condition $\partial A/\partial k = 0$, is shown in Figure 6. The optimum shape here differs very noticeably from those restricted by the condition of fixed drag coefficient.

The minimum attainable value for A for all bodies is 4.84, namely, that of a sphere.

C. Cone-Cylinder with Prescribed Diameter

Here we look briefly at an alternative problem: namely, fixed diameter and drag coefficient, and variable volume. The results of a computation for a sample case are shown in Figure 7, with results for the corresponding prescribed volume problem. It is interesting to note that the two minima occur for approximately the same shape parameters.

IV. PARABOLIC-NOSE PROJECTILE

The tangent-ogive, parabolic-nose projectile has the surface area, correct through terms of order δ^2,

$$S = \pi\ (k\ell)^2\ [\{k^{-1} - (1/3)\}\delta + (\delta^2/4) + (\delta^3/15)] \tag{8}$$

and the volume

$$V = \pi(\delta^2/4)\ (k\ell)^3\ [k^{-1} - 7/15] \tag{9}$$

The constraint, correct through terms of order δ^2, is

$$F \equiv -C_{Do} + c_1\delta^2 + (4C_f/\delta)[\{k^{-1} - (1/3)\} + (\delta^2/15)] + C_{DB} = 0 \qquad (10)$$

Figure 8 presents the variation of k and A with δ for $M_\infty = 1.5$, 2.0, 2.5, and 3.0. In all cases, the minimum A occurs at $k > 1$, so that there is no realizable "optimum" parabolic-nose projectile for the prescribed values of C_{Do}. However, the curves clearly indicate the type of shell shape that one should employ to obtain a low value of A. The smallest A values occur at $k = 1$, and are summarized in the following table:

M_∞	--	1.5	2.0	2.5	3.0
$\delta(k=1)$	--	.335	.315	.306	.290
A	--	6.10	6.22	6.27	6.38

A set of shell shapes fulfilling the condition of Eq. (10) for $M_\infty = 2.0$ is shown in Figure 9. Clearly, the more "efficient" shapes (smaller A) have the smaller values of ℓ/d.

V. CONCLUDING REMARKS

In the foregoing study, linear aerodynamics and a base pressure coefficient independent of shape have been used in the analysis. Also, the particular choice of cone-cylinder and parabolic-nose shell families is one of convenience and the graphs are only a guide for the "optimum shell." Nevertheless, one can draw tentative conclusions from these results; namely i) the shell of minimum surface is a small ℓ/d shell (~3-4) for fixed volume and drag coefficient, and ii) the optimum shape in supersonic flight is nearly independent of Mach number.

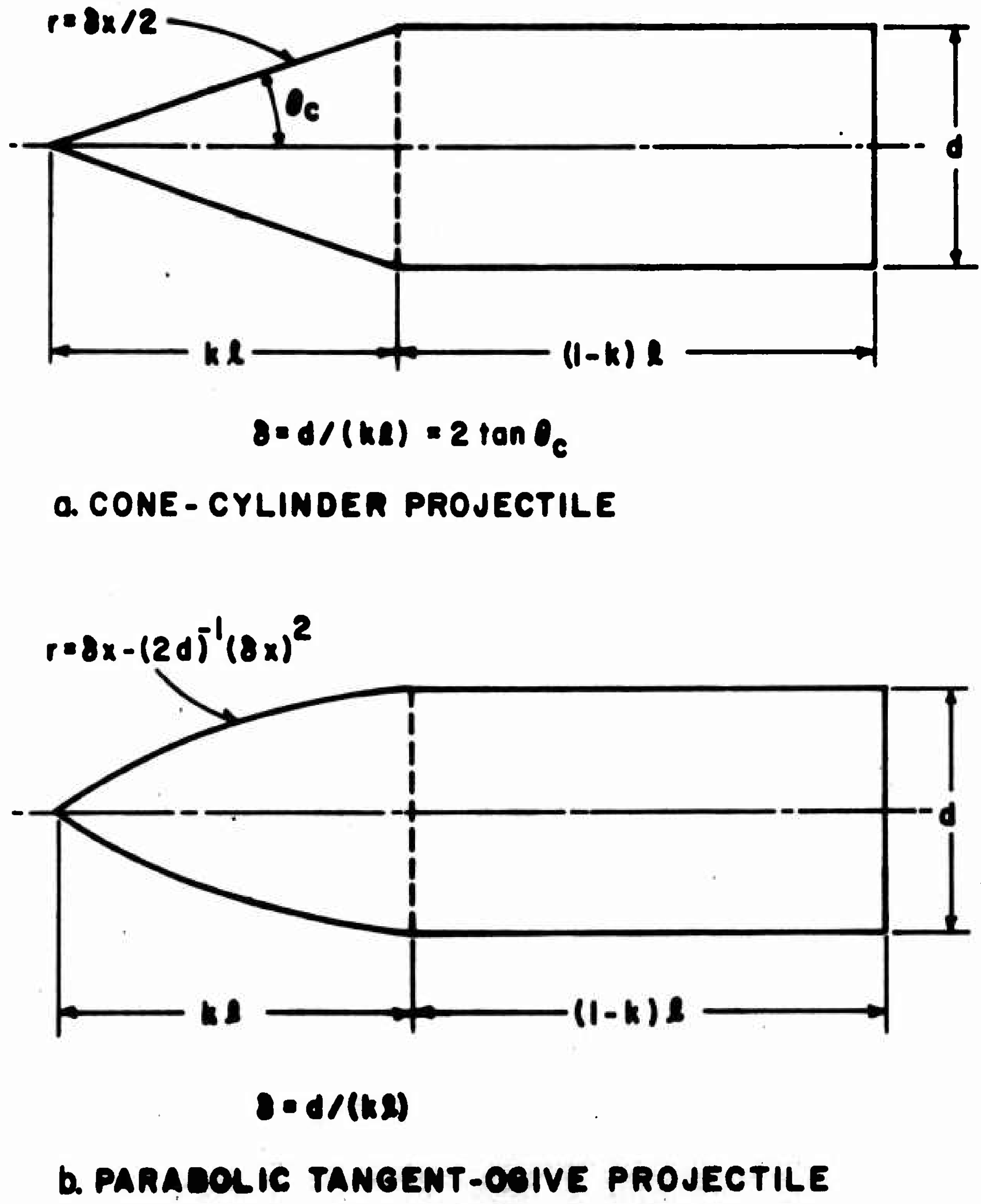
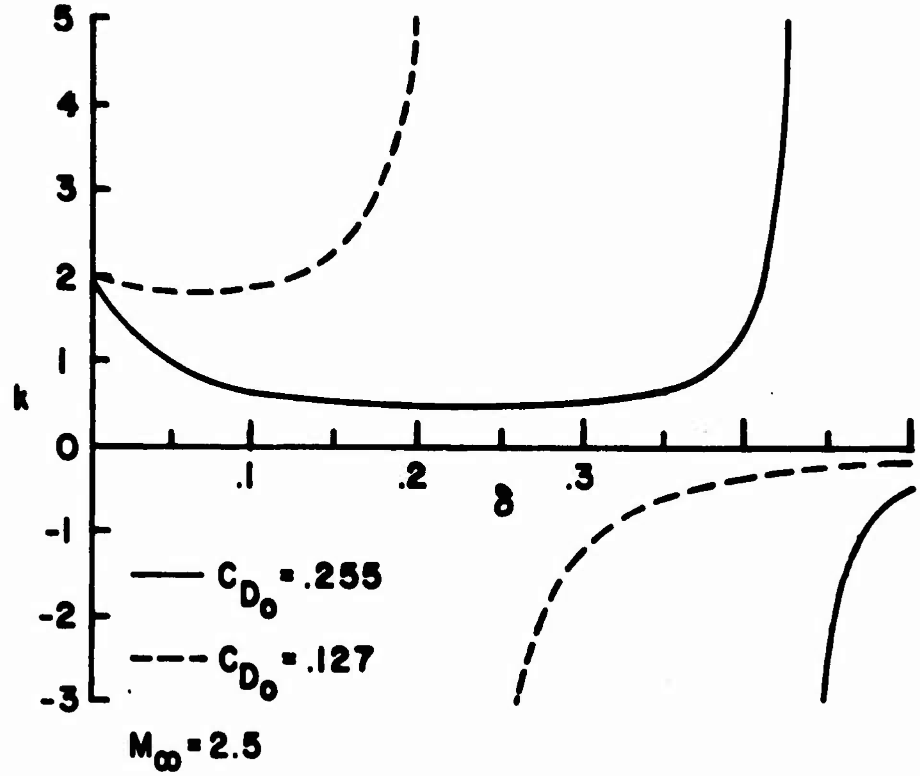
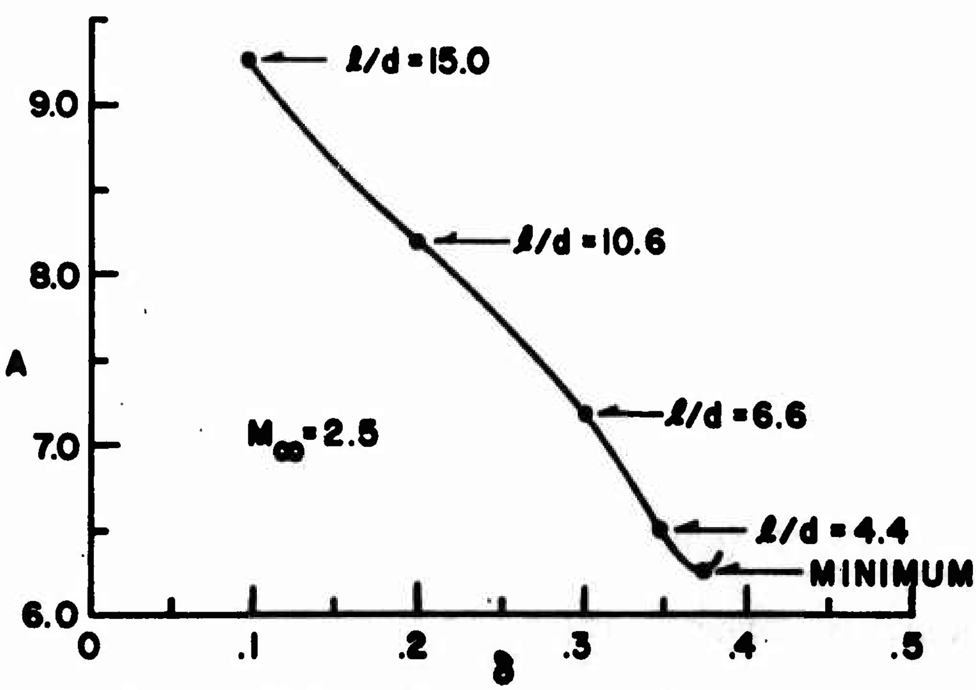


Figure 1. Diagrams of Cone-Cylinder and Parabolic-Nose Tangent-Ogive Projectiles



a. VARIATION OF k -- EQUATION (6)



b. VARIATION OF A

Figure 2. Variation of k and A with δ for Cone-Cylinders with C_{D_o} = .255 (M_∞ = 2.5).

Fig. 2

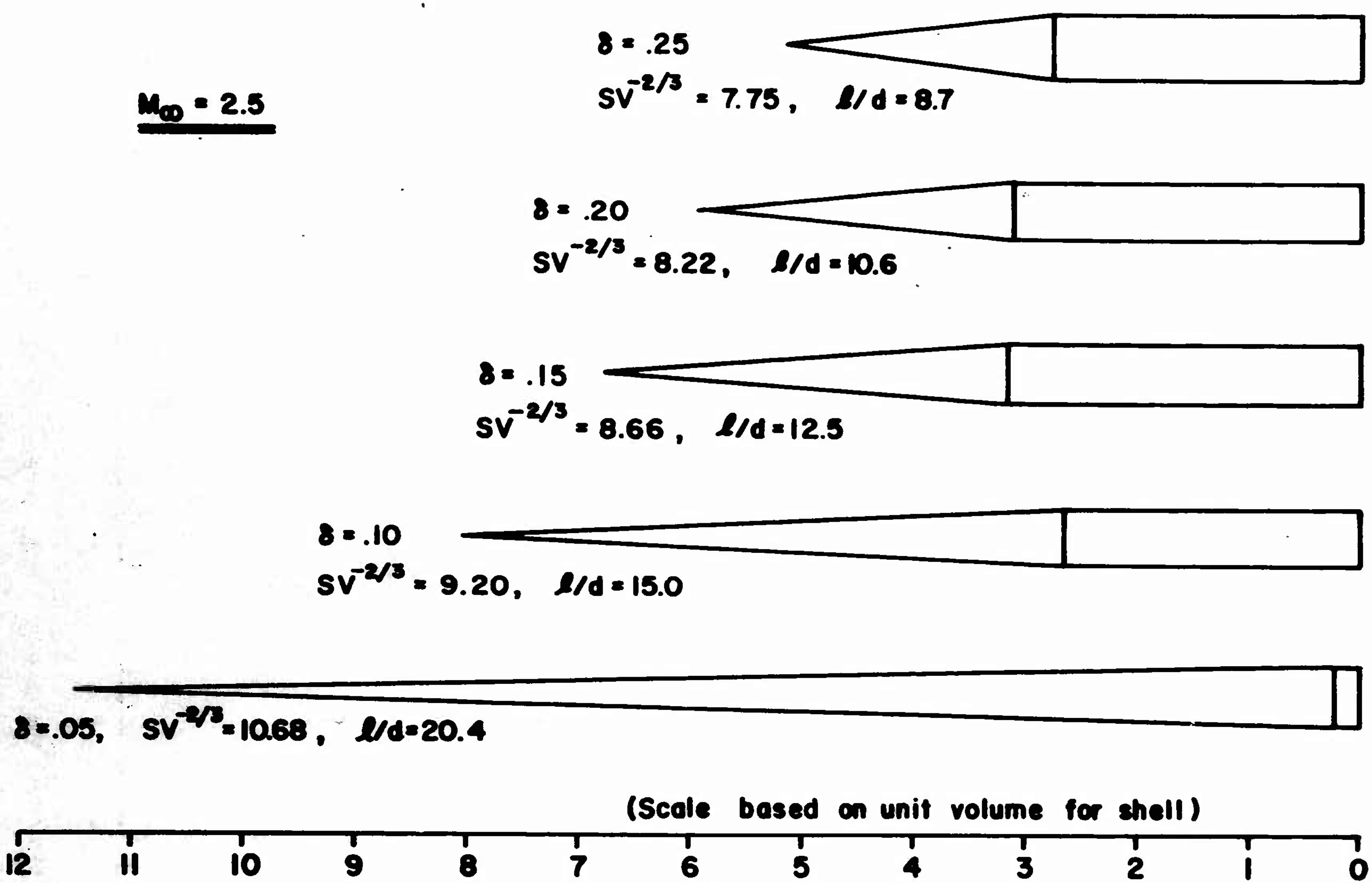


Figure 3. Series of Cone-Cylinders of Unit Volume with Drag Coefficient C_{D_o} = .255 (M_∞ = 2.5).

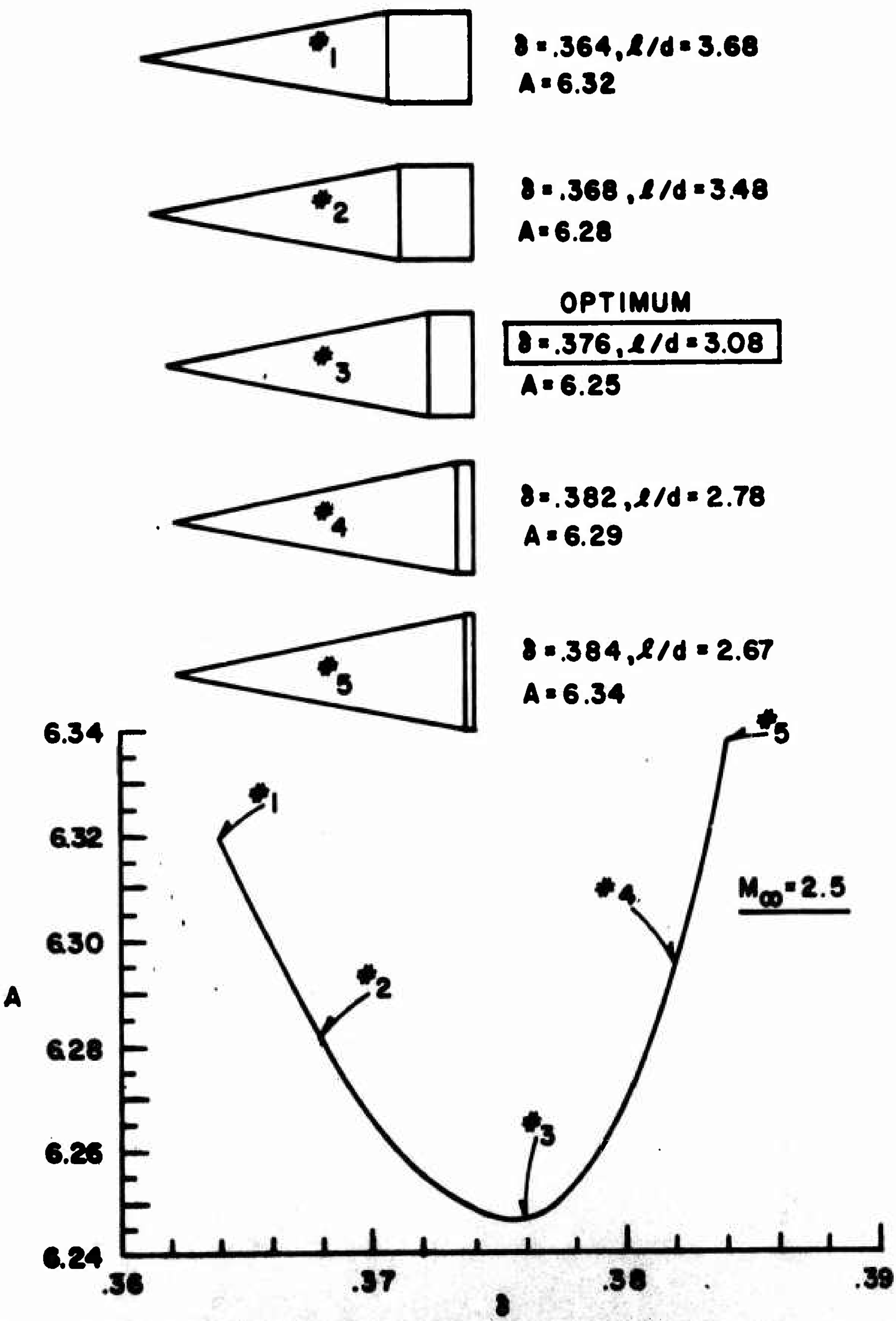


Figure 4. Variation of A with δ Near Its Minimum (for C_{Do} = .255) and Diagrams of Five Cone-Cylinders in This Region.

M_∞ --	1.5	2.0	2.5	3.0
$C_{D_{o\,min}}$ --	.250	.206	.174	.155

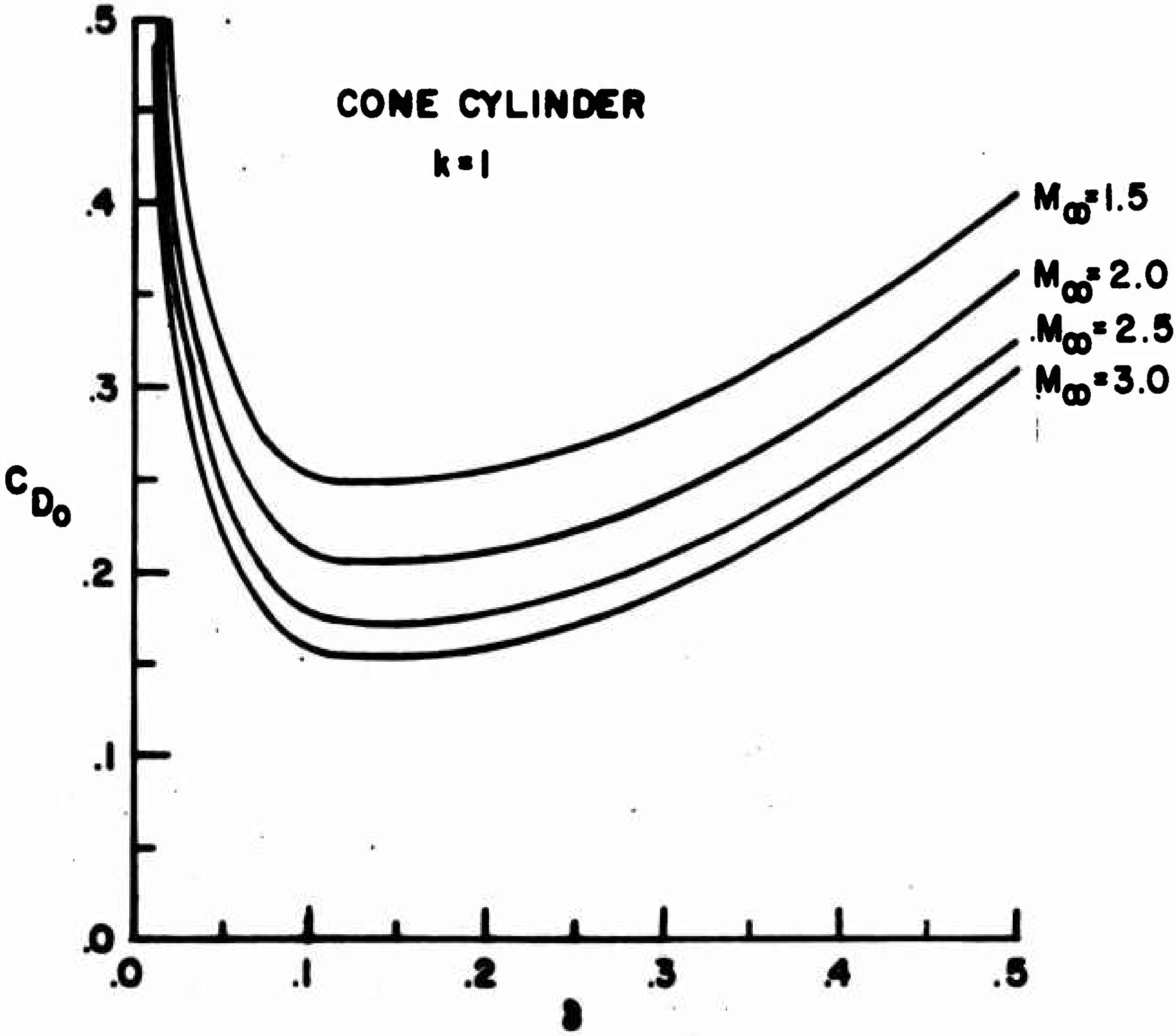


Figure 5. Variation of C_{D_o} with δ and M_∞.

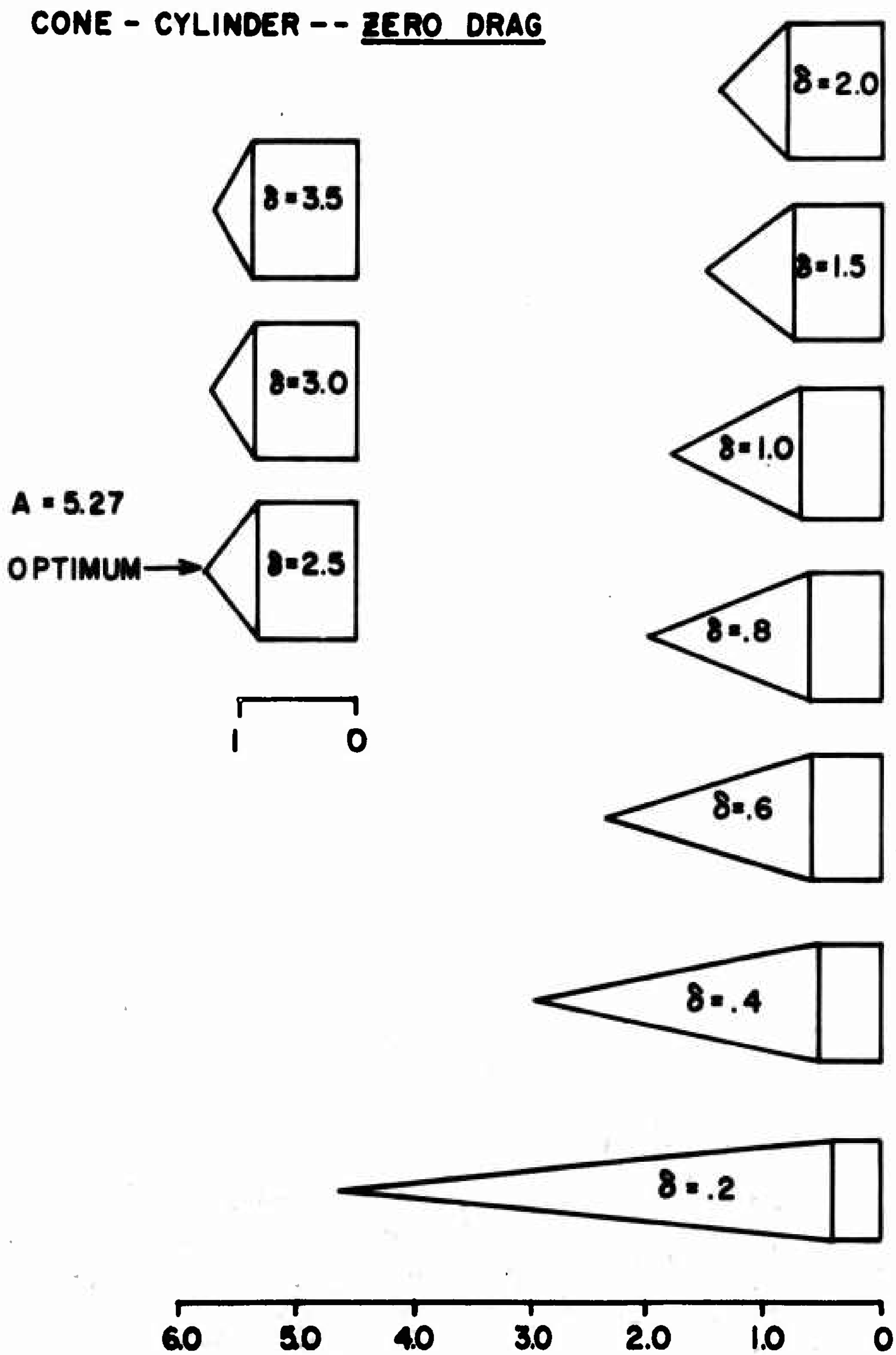


Figure 6. Series of Cone-Cylinders Fulfilling Condition $\partial(A)/\partial k = 0$, with $C_{D_o} = 0$.

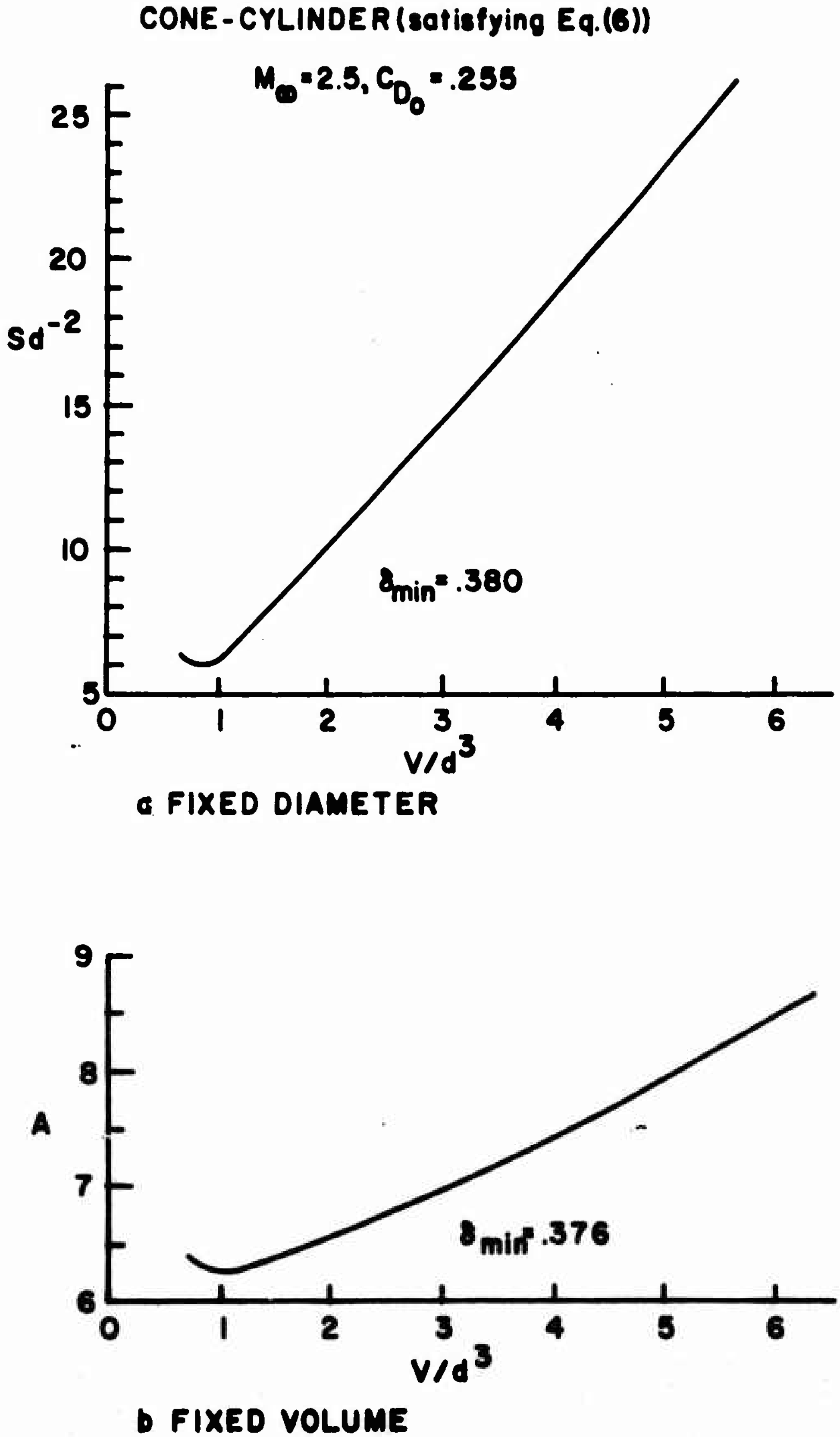


Figure 7. Variation of Cone-Cylinder Surface for Fixed Volume and Fixed Diameter.

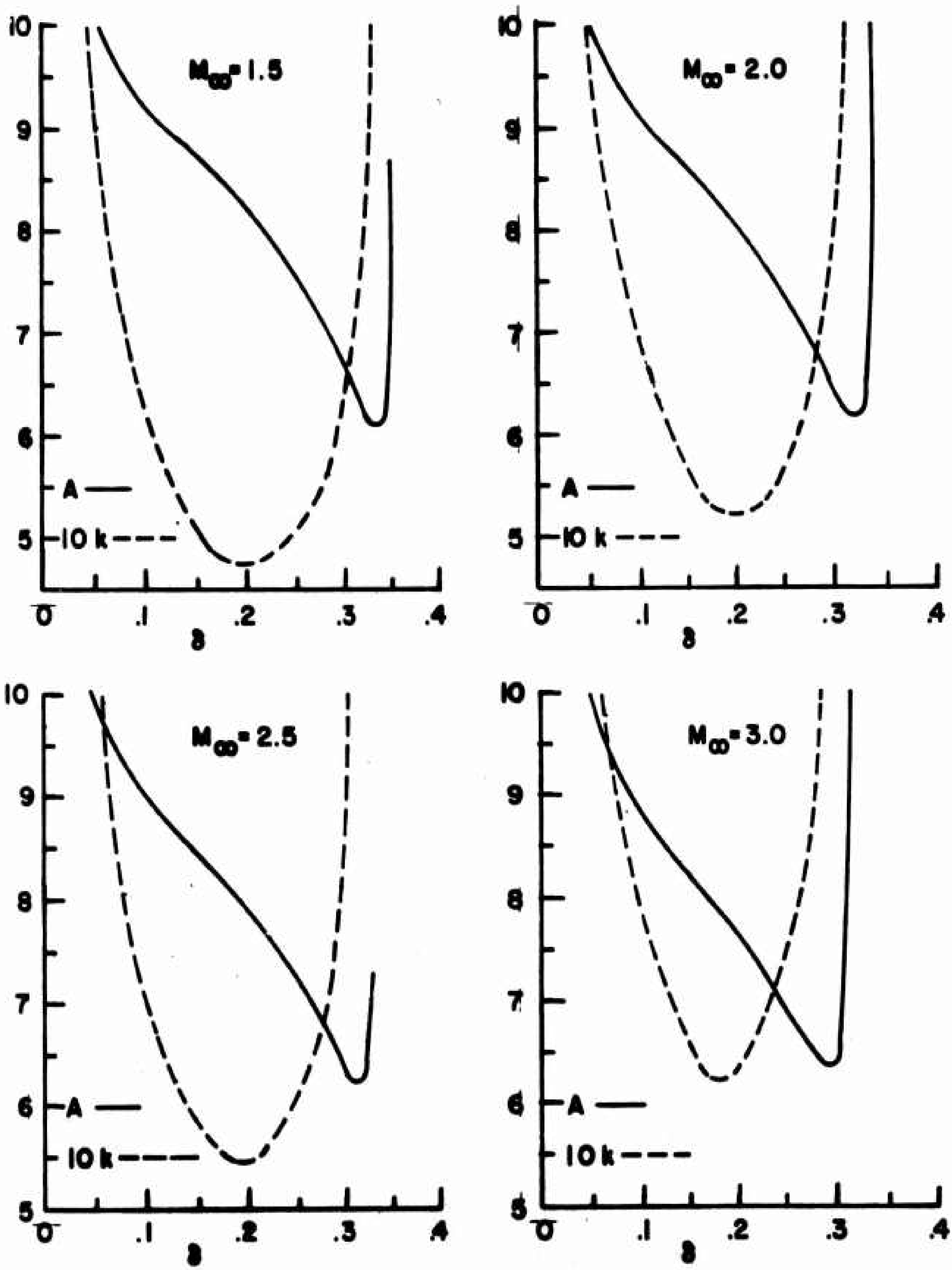


Figure 8. Variation of k and A with δ for Parabolic-Nose Tangent-Ogive Projectiles for Which Eq. (10) is Satisfied.

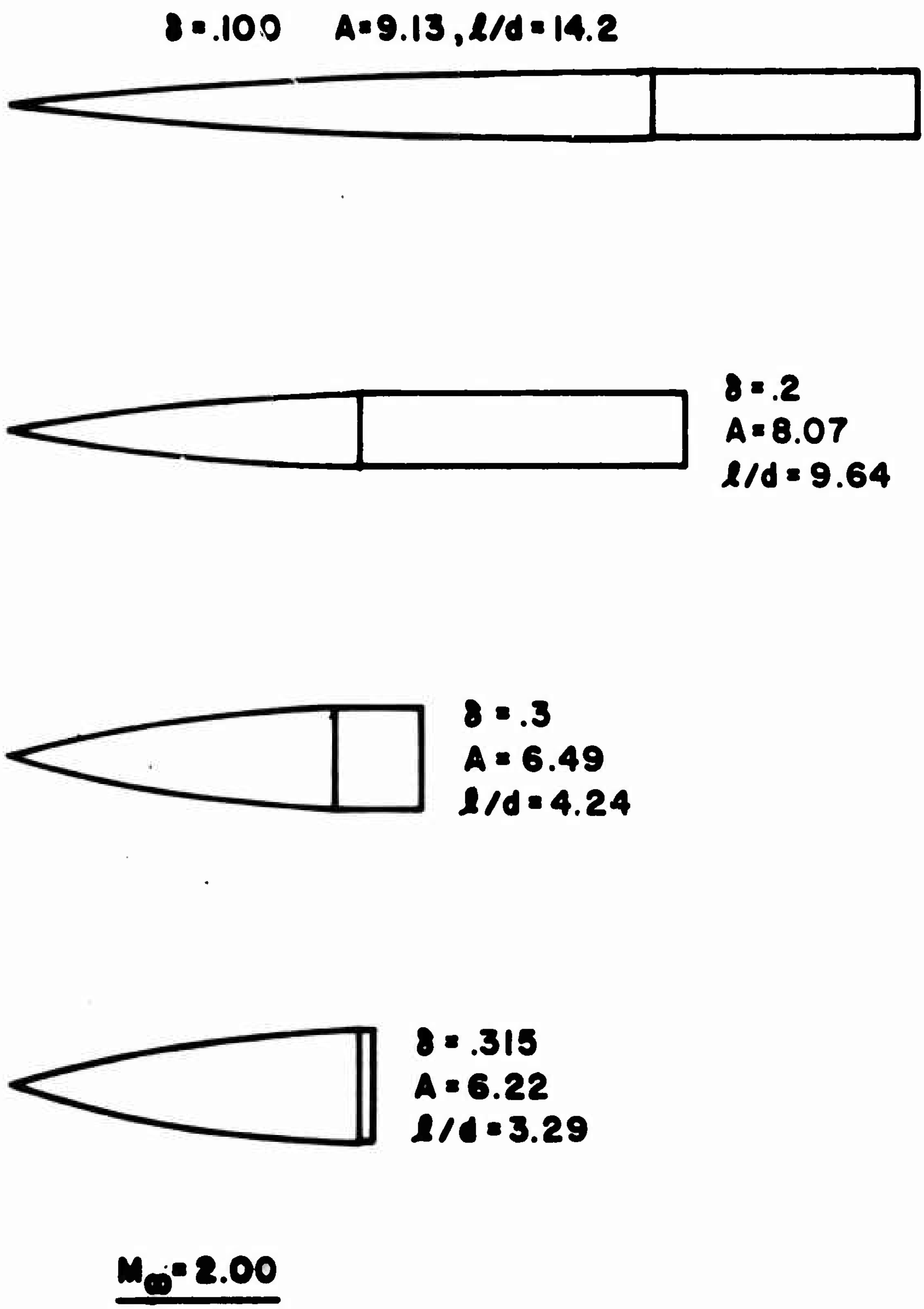


Figure 9. Series of Unit Volume Parabolic-Nose Tangent-Ogive Projectiles for C_{D_o} = .293 (M_∞ = 2.0).

REFERENCES

1. A. C. Charters and R. A. Turetsky, "Determination of Base Pressure from Free Flight Data," Ballistic Research Laboratories Report No. 653, AD No. ATI 26679L, March 1948.

2. H. Schlichting, Boundary Layer Theory, McGraw-Hill Book Company, New York (1960) (p. 541).

3. Z. Kopal, Tables of Supersonic Flow Around Cones, Massachusetts Institute of Technology, Technical Report No. 1 (1947).

4. Handbook of Supersonic Aerodynamics, Vol. 3, Sec. 8, "Bodies of Revolution," Report 1488, Dept. of Navy, Bureau of Naval Weapons, Oct. 1961.

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13. ABSTRACT

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Optimum shell shape						
Improved Ammunition						
Projectile design						
Artillery shell						
Cone-Cylinder Projectile						
Tangent-Ogive Projectile						
Fixed Volume and C_D Projectile						

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